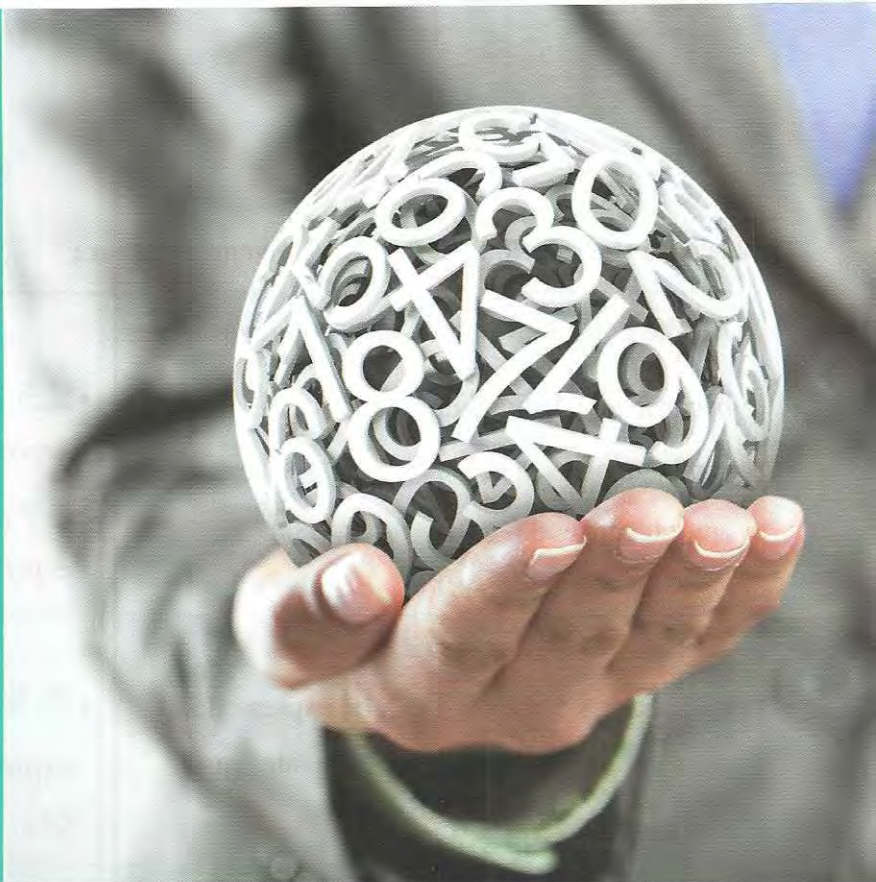


Lesson

1

Set of Rational Numbers



Prelude

- You studied in the primary stage some sets of numbers as :
 - * Set of **COUNTING** numbers = $\{1, 2, 3, 4, \dots\}$
 - * Set of **NATURAL** numbers $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$
 - * Set of **INTEGERS** $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- In this unit , you will recognize another set of numbers called

"**The set of rational numbers**" and it is denoted by the symbol " \mathbb{Q} "

Rational numbers

The numbers : $\frac{1}{2}$, $-\frac{5}{8}$, 3 , 0 , $3\frac{1}{2}$, 0.7 , 2.5 and 15% are **rational numbers**.



Definition of the rational number

A rational number is a number that can be expressed in the form of a quotient of an integer divided by an integer other than 0

i.e. The rational numbers are all numbers can be expressed as $\frac{a}{b}$
where a and b are integers , $b \neq 0$

, where a and b are called the two terms of the rational number $\frac{a}{b}$

So , we can express the set of rational numbers as the following :

The set of rational numbers $\mathbb{Q} = \{x : x = \frac{a}{b} , a \in \mathbb{Z} , b \in \mathbb{Z} , b \neq 0\}$

Based on the previous definition , we can say that :

Examples :

1 All the decimal numbers are rational numbers.

because any decimal number or decimal fraction can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

- 2.5 is a rational number can be expressed in the form $\frac{25}{10}$ or $\frac{250}{100}$ or...
- 0.7 is a rational number can be expressed in the form $\frac{7}{10}$ or $\frac{70}{100}$ or...

2 All percents are rational numbers.

because any percentage can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

- 15 % is a rational number can be expressed in the form $\frac{15}{100}$ or $\frac{150}{1000}$ or...

3 All integers are rational numbers.

because any integer can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Therefore :

The set of integers is a subset of the set of rational numbers.

i.e. $\mathbb{Z} \subset \mathbb{Q}$ and since $\mathbb{N} \subset \mathbb{Z}$
, then $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

and the following figure shows that.



- 3 is a rational number can be expressed in the form $\frac{3}{1}$ or $\frac{6}{2}$ or $\frac{9}{3}$ or ...
- 0 is a rational number can be expressed in the form $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ or ...
- -16 is a rational number can be expressed in the form $-\frac{16}{1}$ or $-\frac{32}{2}$ or $-\frac{48}{3}$ or ...

! Remark

Each integer is a rational number , but not each rational number is an integer.

For example:

- $\frac{12}{6}$ expresses an integer because : 12 is divisible by 6 and the result is 2
- $\frac{25}{4}$ does not express an integer because : 25 is not divisible by 4

**Example 1**

Show why each of the following is a rational number :

1 $3\frac{2}{5}$

2 -0.17

3 0.006

4 27%

Solution

Each of the previous numbers is a rational number because each of them can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$ as follows :

1 $3\frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$

2 $-0.17 = -\frac{17}{100}$

3 $0.006 = \frac{6}{1000}$

4 $27\% = \frac{27}{100}$

Remark

If $\frac{a}{b}$ is a rational number , then $b \neq 0$

Example 2

Choose the correct answer from the given ones :

1 The number $\frac{5}{2x} \in \mathbb{Q}$, if $x \neq \dots\dots\dots$

- (a) 5 (b) 2 (c) 0 (d) -2

2 If $\frac{x-1}{x+3}$ is a rational number , then $x \neq \dots\dots\dots$

- (a) -3 (b) -1 (c) 1 (d) 3

3 The number $\frac{3x-12}{2x-4}$ is not rational , if $x = \dots\dots\dots$

- (a) -2 (b) 0 (c) 2 (d) 4

Solution

1 (c)

The reason : $\frac{5}{2x} \in \mathbb{Q}$ if $2x \neq 0$ i.e. $x \neq 0$

2 (a)

The reason : since $\frac{x-1}{x+3}$ is a rational number
 , then $x+3 \neq 0$ i.e. $x \neq -3$

3 (c)

The reason : $\frac{3x-12}{2x-4} \notin \mathbb{Q}$ if $2x-4=0$
i.e. $2x=4$ i.e. $x = \frac{4}{2} = 2$

TRY 1 by yourself

Complete the following table :

The number	$\frac{5}{x-3}$	$\frac{3}{4-x}$	$\frac{7}{8x}$	$\frac{6x}{x}$
Expresses a rational number if $x \neq$

! Remark

If the rational number $\frac{a}{b} = 0$, then $a = 0$

Example 3

If the rational number $\frac{x-3}{x+3} = 0$, find the value of x

Solution

Since $\frac{x-3}{x+3} = 0$

, therefore $x - 3 = 0$

i.e. $x = 3$

TRY 2 by yourself

Complete the following table :

The rational number	$\frac{x-2}{x-1}$	$\frac{6-x}{x-4}$	$\frac{2x}{x+5}$	$\frac{2x-4}{x+3}$
Equals zero if $x =$

Positive and negative rational number

The rational number $\frac{a}{b}$ is

positive

if the product of its terms is positive

i.e. $a \times b > 0$

and a, b have the same sign.

Examples for positive rational numbers :

$$\bullet \frac{3}{5} \quad \bullet \frac{-2}{-3}$$

equal to zero

if its numerator is zero

i.e. $a = 0$

notice that zero is not positive nor negative.

Examples for rational numbers equal to zero :

$$\bullet \frac{0}{4} \quad \bullet \frac{0}{-2}$$

negative

if the product of its terms is negative

i.e. $a \times b < 0$

and a, b have different signs.

Examples for negative rational numbers :

$$\bullet \frac{-3}{4} \quad \bullet \frac{2}{-7} \quad \bullet -\frac{4}{5}$$



i.e. The set of rational numbers $\mathbb{Q} = \mathbb{Q}_+ \cup \{0\} \cup \mathbb{Q}_-$

Where \mathbb{Q}_+ is the set of positive rational numbers, \mathbb{Q}_- is the set of negative rational numbers.

Note that : $\mathbb{Q}_+ \cap \mathbb{Q}_- = \emptyset$

TRY 3 by yourself

Show which of the following numbers is positive, which is negative and which is zero :

$$\frac{3}{4}, \frac{-2}{-9}, \frac{\text{zero}}{-5}, \left| -\frac{1}{2} \right|, \frac{-7}{11}, (-5)^2$$

Different forms of a rational number

The rational number $\frac{a}{b}$ can be written in the form of another rational number $\frac{c}{d}$ equal to it by applying the following property :

Property

The value of the rational number $\frac{a}{b}$ does not change if its two terms are multiplied or divided by an integer \neq zero.

For example:

$$\bullet \frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{6}{14}, \quad \frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$$

$$\frac{3}{7} = \frac{6}{14} = \frac{9}{21}$$

i.e. $\frac{3}{7}, \frac{6}{14}, \frac{9}{21}$ are different forms which represent **the same number**.

$$\bullet \frac{24}{36} = \frac{24 \div 2}{36 \div 2} = \frac{12}{18}, \quad \frac{24}{36} = \frac{24 \div 4}{36 \div 4} = \frac{6}{9}$$

$$\frac{24}{36} = \frac{12}{18} = \frac{6}{9}$$

i.e. $\frac{24}{36}, \frac{12}{18}, \frac{6}{9}$ are different forms which represent **the same number**.

TRY 4 by yourself

Write in three other forms each of the following rational numbers :

1 $\frac{2}{3}$

2 $\frac{16}{64}$

Writing a rational number $\frac{a}{b}$ in its simplest form

For any rational number expressed as $\frac{a}{b}$, we say that this rational number is in its simplest form if each of its terms has the smallest possible value.

For example:

- The simplest form of the rational number $\frac{16}{32}$ is $\frac{1}{2}$ and note that : $\frac{16}{32}$ and $\frac{1}{2}$ represent the same rational number.
- The rational number $\frac{3}{14}$ is in its simplest form and can not be simplified to more simple form.

$$\frac{16}{32} = \frac{1}{2}$$

So, they represent the same rational number.

“To put a rational number $\frac{a}{b}$ in its simplest form ,
divide each of its terms by the highest common factor (H.C.F.) between them.”

Example 4 Put each of the following numbers in its simplest form :

1 $\frac{8}{12}$

2 $-\frac{12}{36}$

Solution

- 1 The (H.C.F.) of 8 and 12 is 4

Dividing the two terms of $\frac{8}{12}$ by 4, we get : $\frac{8}{12} = \frac{2}{3}$

- 2 The (H.C.F.) of 12 and 36 is 12

Dividing the two terms of $-\frac{12}{36}$ by 12, we get : $-\frac{12}{36} = -\frac{1}{3}$

TRY
by yourself

5

Complete the following table :

The number	$\frac{5}{25}$	$-\frac{6}{9}$	$\frac{27}{45}$	$-\frac{12}{30}$
Its simplest form

Writing the rational number in the form of percentage

To write the rational number in the form of percentage we express it as $\frac{a}{100}$
which means a %

Example 5 Write each of the following numbers in the form of percentage :

1 $\frac{9}{20}$

2 $\frac{5}{16}$

3 $\frac{17}{1000}$

4 $5\frac{12}{125}$

5 3.2

**Solution**

$$1 \quad \frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 45\%$$

Another solution : $\frac{9}{20} = \frac{\frac{9}{20} \times 100}{100} = \frac{45}{100} = 45\%$

$$2 \quad \frac{5}{16} = \frac{\frac{5}{16} \times 100}{100} = \frac{31.25}{100} = 31.25\%$$

$$3 \quad \frac{17}{1000} = \frac{\frac{17}{1000} \times 100}{100} = \frac{1.7}{100} = 1.7\%$$

$$4 \quad 5\frac{12}{125} = \frac{637}{125} = \frac{\frac{637}{125} \times 100}{100} = \frac{509.6}{100} = 509.6\%$$

$$5 \quad 3.2 = \frac{32}{10} = \frac{32 \times 10}{10 \times 10} = \frac{320}{100} = 320\%$$

TRY
by yourself **6**

Write each of the following numbers in the form of percentage :

$$1 \quad \frac{4}{5}$$

$$2 \quad \frac{3}{1000}$$

$$3 \quad 2.5$$

Changing a rational number from the form $\frac{a}{b}$ to a decimal form

Some rational numbers could be changed from the form $\frac{a}{b}$ into a terminating decimal.

For example: • The rational number $\frac{3}{5}$ can be changed into 0.6

• The rational number $\frac{3}{2}$ can be changed into 1.5

“To write a rational number in the form of a terminating decimal, make its denominator equal to 10, 100, 1000 or ...”

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$$

$$\frac{3 \times 5}{2 \times 5} = \frac{15}{10} = 1.5$$

Example 6

Write each of the following numbers in the form of a terminating decimal :

$$1 \quad \frac{2}{5}$$

$$2 \quad |-\frac{3}{8}|$$

$$3 \quad -2\frac{7}{25}$$

Solution

$$1 \quad \frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$$

$$2 \quad |-\frac{3}{8}| = \frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$$

$$3 \quad -2\frac{7}{25} = -2\frac{7 \times 4}{25 \times 4} = -2\frac{28}{100} = -2.28$$



Check your
answer using
calculator

TRY 7
by yourself

Write each of the following rational numbers in the form of a terminating decimal :

1 $\frac{3}{4}$

2 $\frac{11}{20}$

Remark

Some rational numbers could not be changed into terminating decimal as the rational number $\frac{1}{3}$, then using calculator, you find that : $\frac{1}{3} = 0.333333 \dots$

We express that as $(0.\dot{3})$ and read it as the infinite repeating decimal 0.3 (the recurring decimal 0.3) where the dot above the digit 3 means the digit is repeating (recurring).

Example 7

Using a calculator, write each of the following rational numbers in the form of a recurring decimal :

1 $\frac{2}{3}$

2 $\frac{2}{11}$

3 $5\frac{71}{333}$

Solution

- 1 Using the calculator, we get that :

$$\frac{2}{3} = 0.6666666667$$

i.e. $\frac{2}{3} = 0.\dot{6}$

- 2 Using the calculator, we get that :

$$\frac{2}{11} = 0.1818181818$$

i.e. $\frac{2}{11} = 0.1\dot{8}$

- 3 Using the calculator, we get that :

$$\frac{71}{333} = 0.2132132132$$

i.e. $5\frac{71}{333} = 5.\dot{2}1\dot{3}$

Notice that :

Putting dots above the first and last digits means repeating all digits (first, last and between them)

$$5.\dot{2}1\dot{3}$$

TRY 8
by yourself

Write each of the following rational numbers in the form of a recurring decimal :

1 $\frac{3}{11}$

2 $\frac{41}{333}$



! Remark

It is possible to write the recurring decimal in the form of $\frac{a}{b}$ by using scientific calculators of type CASIO *fx-95ES* plus or a different type.

Notice that some scientific calculators can not be able to solve this problem.

For example:

- To write the number $0.\dot{2}\dot{1}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.21212121212121, then press **=** you will get the rational number $\frac{7}{33}$
- To write the number $0.1\dot{3}\dot{6}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.1363636363636, then press **=** you will get the rational number $\frac{3}{22}$



TRY **9** by yourself

Use the calculator to write each of the following in the form $\frac{a}{b}$:

1 $0.\dot{1}\dot{5}$

2 $0.14\dot{5}$

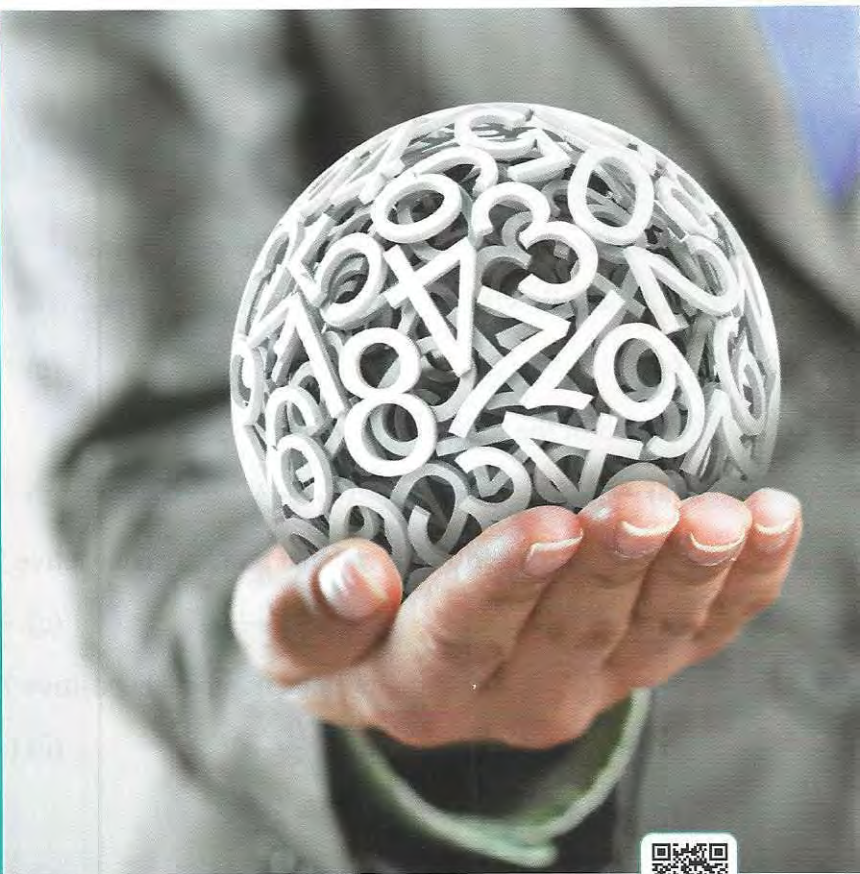


Exercise

1

Set of Rational Numbers

From the school book



● Remember

● Understand

● Apply

● Problem Solving



Interactive test

1 Complete the following :

- 1 If $\frac{5}{a}$ is a rational number, then $a \neq \dots\dots\dots$
- 2 The necessary condition to be $\frac{3}{x-2}$ is a rational number is $x \neq \dots\dots\dots$
- 3 The number $\frac{2}{3x} \in \mathbb{Q}$ if $x \neq \dots\dots\dots$
- 4 The number $\frac{x-3}{3x+6}$ is a rational number if $x \neq \dots\dots\dots$
- 5 The number $\frac{a-6}{a-4}$ is not rational if $a = \dots\dots\dots$
- 6 The rational number $\frac{x-5}{x} = 0$ if $x = \dots\dots\dots$
- 7 The rational number $\frac{4-x}{x-3} = 0$ if $x = \dots\dots\dots$
- 8 The rational number $\frac{5x+15}{x-5} = 0$ if $x = \dots\dots\dots$
- 9 If $\frac{x+4}{x-3}$ is not rational, then $x-2 = \dots\dots\dots$
- 10 $\frac{3}{4} = \frac{9}{\dots\dots\dots}$
- 11 $-\frac{16}{20} = \frac{\dots\dots\dots}{10}$
- 12 $\frac{7}{20} = \dots\dots\dots \%$
- 13 $\frac{21}{1000} = \dots\dots\dots \%$
- 14 $|-0.4| = \dots\dots\dots \%$

2 Choose the correct answer from the given ones :

- 1 All the following numbers are rational except
 - (a) 0
 - (b) $\frac{2}{5}$
 - (c) $\frac{3-3}{7}$
 - (d) $\frac{4}{5-5}$
- 2 Which of the following numbers is an integer ?
 - (a) $-\frac{24}{5}$
 - (b) $\frac{4}{8}$
 - (c) $\frac{15}{5}$
 - (d) $3\frac{1}{4}$
- 3 Which of the following rational numbers is negative ?
 - (a) $\frac{0}{-3}$
 - (b) $-|-\frac{1}{2}|$
 - (c) $\frac{-3}{-4}$
 - (d) $(-7)^2$
- 4 Which of the following rational numbers is positive ?
 - (a) $-\frac{3}{4}$
 - (b) $\frac{0}{5}$
 - (c) $(-5)^3$
 - (d) $\frac{-2}{-9}$
- 5 Which of the following equals $\frac{4}{5}$?
 - (a) 4 %
 - (b) 54 %
 - (c) 120 %
 - (d) 80 %
- 6 If $-\frac{4}{5} = \frac{20}{x}$, then $x =$
 - (a) 25
 - (b) -25
 - (c) 5
 - (d) 100
- 7 The rational number $\frac{a}{b}$ is positive if
 - (a) $a \cdot b > 0$
 - (b) $a \cdot b < 0$
 - (c) $a + b = 0$
 - (d) $a > b$
- 8 The rational number $\frac{-7}{a}$ is positive if a zero
 - (a) $>$
 - (b) \geq
 - (c) $<$
 - (d) $=$
- 9 The rational number $\frac{x}{-5}$ is negative if x zero
 - (a) $>$
 - (b) $<$
 - (c) \leq
 - (d) $=$
- 10 If $a = 2$, $b = 6$, then which of the following is not a rational number ?
 - (a) $\frac{b}{a}$
 - (b) $-\frac{2}{a}$
 - (c) $\frac{0}{a+b}$
 - (d) $\frac{2 \cdot b}{a-2}$
- 11 $0.\dot{5}\dot{7} =$
 - (a) $\frac{57}{100}$
 - (b) $\frac{75}{99}$
 - (c) $\frac{575}{1000}$
 - (d) $\frac{19}{33}$
- 12 $|- \frac{8}{25}| =$
 - (a) $-\frac{8}{25}$
 - (b) $-0.\dot{3}\dot{2}$
 - (c) $0.\dot{3}\dot{2}$
 - (d) 32 %
- 13 12 % =
 - (a) $0.\dot{3}$
 - (b) 1.2
 - (c) $\frac{3}{25}$
 - (d) 0.012

3 Put each of the following numbers in the simplest form :

1 $\frac{15}{25}$

2 $-\frac{24}{56}$

3 $\frac{45}{20}$

4 $-\frac{132}{88}$

4 Which of the following rational numbers can be written as a terminating decimal ?

1 $\frac{7}{15}$

2 $\frac{7}{20}$

3 $\frac{5}{8}$

4 $-\frac{8}{9}$

5 $\frac{5}{11}$

6 $-\frac{13}{22}$

7 $\frac{17}{6}$

8 $2\frac{2}{5}$

9 $-1\frac{2}{3}$

10 $|-1\frac{2}{9}|$

5 Write each of the following two numbers in the form of a recurring decimal :

1 $\frac{6}{11}$

2 $-3\frac{1}{15}$

6 Write each rational number in the form $\frac{a}{b}$:

1 -5

2 zero

3 0.75

4 -0.01

5 5.4

6 30%

7 4.5%

8 $8\frac{2}{3}$

7 Write each of the following rational numbers as a decimal and a percentage :

1 $2\frac{1}{2}$

2 $-\frac{3}{20}$

3 $7\frac{3}{16}$

4 $\frac{1}{6}$

8 Why does the definition of a rational number $\frac{a}{b}$ state that $b \neq 0$?

For excellent pupils

9 Choose the correct answer from the given ones :

1 If $\frac{a}{b}$ is a rational number and $ab = \text{zero}$, then

(a) $a = 0, b \neq 0$

(b) $a \neq 0, b \neq 0$

(c) $a = 0, b = 0$

(d) $a \neq 0, b = 0$

2 The number $\frac{5x}{|x|-2} \notin \mathbb{Q}$ if $x = \dots\dots\dots$

(a) zero

(b) -1

(c) ± 2

(d) 5

10 Write the rational number $\frac{a}{b}$ that equals $\frac{3}{5}$ and the sum of its two terms is 24

11 If $x \in \mathbb{N}$, find the values of x which make each of the following an integer :

1 $\frac{75}{x}$

2 $\frac{15}{x+1}$